Algebraic Expressions

Monomial : is an algebraic expression with 1 term. It can be:

- *A variable*: $a \quad ; \quad x \quad ; \quad t$
- *A constant*: $5 \quad ; \quad -3 \quad ; \frac{1}{2}$
- *A product:* $2a \quad ; \quad -4x^2 \quad ; \quad 3xy \quad ; \quad \frac{1}{2}x^2y$

Note: the exponent must be a non-negative integer. i.e. $3x^{-2} \quad ; \quad 2\sqrt{x} \quad ; \quad 5x^{1/3}$ are not monomials

Coefficient: is the factor by which a variable is multiplied

$$3x^n \rightarrow \text{exponent} \in \mathbb{N}$$
$$\therefore \ddots$$
$$\text{coefficient} \qquad \text{variable}$$

Note: if the coefficient is 1, it is not written for example: ab = 1ab ; $-1x^2 = -x^2$

Like terms: are terms with identical variables and identical exponents (not coefficients)

Examples: 6 and -2 $-2a^3b^2$ and $5a^3b^2$
3a and 4a $0.5xy^5$ and $10xy^5$

The Degree of a term is the sum of the exponents of the variables.

Examples:	3	degree	0	
	3x	degree	1	
	$3x^2$ or 3xy	degree	2	
	$3x^2y$	degree	3	
	$3x^2y^3$	degree	5	etc.

To find the numerical value of an algebraic expression we replace the variable by the given value.

	$4x^3 \quad if x = 2$	$2a^2 \quad if a = -3$	$2x^3y^2 \quad if x = 2; y = -3$
Examples:	$=4(2)^3$	$=2(-3)^2$	$=2(2)^3(-3)^2$
	$=4(8)$	$=2(9)$	$=2(8)(9)$
	$=32$	$=18$	$=144$

Binomial: is an algebraic expression with 2 terms.

Examples: $3x + 2$; $2a^2 + 3a$; $4ab - 2a$

Trinomial: is an algebraic expression with 3 terms.

Examples: $2a^2 + 3a + 5$; $b^3 - 2b + 5$; $2x^2 - 6xy + 7y$

Polynomial: is an algebraic expression with 1 or more terms, separated by +/-, and the terms are written in decreasing order of powers.

The degree of a polynomial: is the degree of the term with the highest degree.
Example: $3x^2y^2 + 4xy^2$ has degree 4

Simplifying an algebraic expression means representing it using as few terms as possible (collecting like terms)

The **Zero** of a polynomial is the value of the variable which makes the polynomial equal to zero

2.1 Monomials

Refer to first half of the Handout: "Algebraic Expressions", for definitions.

A **MONOMIAL** is the product of a variable with a positive integer exponent and real number.

LIKE TERMS are terms with identical variables and identical exponents (not coefficients)

The **DEGREE** of a monomial is the sum of all its exponents.

$\frac{1}{b^5}$ $\frac{1}{2}y$ $3x$ 7 $\sqrt{5a}$ $12a^{\frac{1}{2}}$ $-22a^5b^7$ $2y^{-5}$

Ex 1: Monomial Vs Not a Monomial

Ex 2: Are the following pairs like terms?

1) 2a , -2a
2) 4b , 6ba
3) $3x$, $-7x^2$
4) abc , -abc
5) $3b^0$, 5
6) $6x$, $\frac{4}{x}$
7) $3x^2y$, $4xy^2$
8) $11st^2u^3$, $9u^3t^2s$
9) $\frac{2}{5}$, -8
10) 2a , 3ab
11) $3x$, $3x^0$
12) $2ax^2$, ax
13) $2a^2x^3$, $-2a^2x^3$
14) Is $2x^{-1}$ a monomial?

Ex 3: Determine the degree of each monomial

Monomial	$5x^2$	$3y^{12}$	-7	$6xy^4$	$3a^3b^3$
Degree					

We can use Algi-tiles to represent single variable polynomials: Introducing the Tiles …….

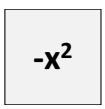
+1 Tile

-1 Tile

+x Bar

-x Bar

+x^2 Square



- x^2 Square

Note that 2 opposites of the same type cancel each other out when added.

Practice:
Page 50 # 1, 2, 3

2.2 Monomial Operations

- **Adding/ Subtracting**: Only like terms can be +/- (simplified to a single term)
  - Non like terms cannot be simplified to a single term
  - When you +/- terms, do it to the **coefficients** only.

- **Multiplying/ Dividing**: they don't have to be like terms.
- **Multiplying:** $(ax^m)(bx^n) = abx^{m+n}$
- **Dividing:** $\frac{ax^m}{bx^n} = \frac{a}{b}x^{m-n}$

Ex 1: Simplify the following monomials

$3x^2 + 4x^2$ _____ Adding

$5x^3y^2 - 3x^3y^2$ _____

$(2a)(5b)$ _____ Multiplying

$4(1.5a)$ _____

$\frac{12x^3y^4}{6x^2y^2}$ _____ Dividing

Practice:
Handout
Page 52 # 1, (2 - 7 aceg each), 8

2.3 Polynomials

Refer to second half of the Handout: "Algebraic Expressions", for definitions.

Do P. 54 Act. 1 and read the green box that follows.

A **POLYNOMIAL** is the sum or difference of many unlike MONOMIALS.

Write the terms in decreasing order of degrees.

Ex: $12x^7 + 6x^4 - 7x^2 + 7$

P(x) is just the notation to name the polynomial

Ex 1: Simplify:

$P(x) = 2x^2 + 5x^3 + 3x + 6 + 3x + 4x^2 + 7 - 5x^3$

Ex 2: Evaluate the above trinomial for x = 2

(ie. Evaluate P(2))

P(2) =

Ex 3: Rewrite each polynomial and give its degree.

a) $4xy^2 + 3x^2y^2$ b) $2 - 5y^2 + 6y$

Ex 4: If $P(x,y) = -3x^2y + 2xy^2 - 2x + 3y - 5$; evaluate

P(-2,1)

Ex 5: A mother is 5 times as old as her daughter.

a) If the girl is x years old, how old is the mother?

b) How old will each be in 13 years?

	Mother	Girl
Now		
In 13 years		

c) What will their total age be in 13 years?

Practice:

Page 55 # 1,2

Page 55 # 3 – 12

(6,9,11,12 all, the rest a,c only)

2.4 Polynomial Operations
-A- Sum and difference of Polynomials

Adding Polynomials: group like terms

Ex 1: $2x^2 - 3x - 3$

$+ \ -2x^2 + 2x + 6$

$=$

1

Ex 2: Simplify

$3x^2 + (5x + 10x) =$ _____

$(8xy^2 + 9x^2y) + 5xy^2 =$ _____

$(6a + 12b) + (7a + 5b) =$ _____

$(7y + 6) + (8y^2 + 10) =$ _____

2

Subtracting polynomials: subtract each like term.
(It is like adding the opposite of each term)

Ex 3: $2a^2 + 5a + 8$

$- \ a^2 - 4a + 5$

$=$

Same as: $2a^2 + 5a + 8$

$+ - a^2 + 4a - 5$

$=$

3

Ex 4: Simplify by subtracting the polynomials

$7x - (5x + 10x) =$ _____

$8x^2 + 4x - (6x^2 + 2x) =$ _____

$4a + 7b - (12a - 5b) =$ _____

$7c + 6c^2 - (8c^2 - 10) =$ _____

4

Ex 5: Sam works at a Telephone Company he gets $16/h but pays $35 per paycheque for insurance. Pam works in a restaurant; she gets $10/h and $120 tips per pay period.

a) Write an algebraic expression for each of them.

b) Find the total of their incomes.

5

S(h) = 16h – 35 **P(h) = 10h + 120**

T(h) = 26h + 85

c) Find the difference between their incomes

d) If in one pay period they work 30 hours each, what is their total pay?

6

Ex 6: Mix bag Polynomials review

1) Simplify: $3x^2 + 10x^2 – 6x + 4x$
2) True or false:
   a) A monomial can have a negative exponent.
   b) Like terms are monomials with the same variables raised to the same exponents.
   c) A polynomial has at least two UNLIKE TERMS.
3) Simplify: $3a + 5b – (7a + 9b)$
4) Is $4x^2 – 7x + 10$ a trinomial?
5) Simplify: $3x + 7y – (2x – 6y)$
6) Circle the monomials

$\sqrt{5a}$ $7a^5b^7$ y^{-10} 6 $12x^4$

Solutions

1) $13x^2 - 2x$
2) a) False
   b) True
   c) True
3) $-4a - 4b$
4) Yes
5) $x + 13y$
6) $7a^5b^7$, 6 , & $12x^4$

Practice:
Page 57 # 1(aceg), 2(ac), 3, 4, 5

2.4 Polynomial Operations
-B- Product of Polynomials

Case 1- Monomial times a Polynomial:

Examples: 3(2x + 4) =

Distributive Property

Page 59 # 6 (a) 3x(5x – 2) =

(b) $-2x^2(3x + 5)$ =

(f) $\frac{2}{3}x^2(6x^2 - 9x + 3)$ =

Ex 1: Multiplying Polynomials

$3x(5x^2 + 2x) =$ _____

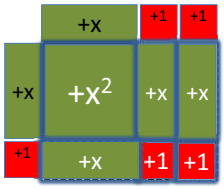
$-5(2x+1) =$ _____

$-2x^2(3x-3) =$ _____

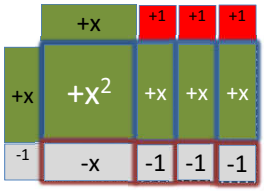
$\frac{3}{2}x^2(6x^3 - 8x + 4) =$ _____

Case 2- Binomial times a Binomial:

a) (x + 1)(x + 2)



b) (x - 1)(x + 3)



=

Case 2- Binomial times a Binomial:

$(2x + 2)(x + 4)$

Option 1: Use Distributive Property:

$2x(x + 4) + 2(x + 4)$

= $2x^2 + 8x + 2x + 8$

= $2x^2 + 10x + 8$

Option 2: Expand and simplify (FOIL)

$(2x + 2)(x + 4)$

F - First
O - Outside
I - Inside
L - Last

= (2x)(x) + (2x)(4) + (2)(x) + (2)(4)

= $2x^2$ + 8x + 2x + 8

= $2x^2 + 10x + 8$

Ex 2: Expand and simplify (FOIL)

$(x - 3)(x + 5)$

Ex 3: Foil practice

$(x+2)(x+2) =$ _____

$(x-3)^2 =$

_____

$(4x-3)(2x+1) =$ _____

Practice:
Page 59 # (6, 7, 8, 9 aceg of each)
page 60 # 11, 12

Ex 4: (page 59 # 8 (d))
(3x + 2)(2x – 3) – (x – 1)(2x + 1)

Ex 5: Find the missing factor

(_____)(2x – 5) = $8x^2 - 20x$

Practice:
Page 60 # 13--22

2.4 Polynomial Operations
-D- Division of Polynomials

Case 1- Polynomial divided by a constant:

Ex 1: $(4x + 8) \div 2 =$

Ex 2: $\frac{6x - 18}{3} =$

Ex 3: $\frac{8x^2 + 12x + 16}{4} =$

Divide each term by the constant

Case 2- Polynomial divided by a Monomial:

Ex 1: $(4x^2 + 2x) \div 2x =$

Ex 2: $\frac{-14x^3 + 35x^2}{7x^2} =$

Ex 3: $\frac{4x^3 + 8x^2 - 6x}{2x} =$

Divide each term by the Monomial. Following the laws of exponents.

Ex 4: Simplify by dividing

$\frac{9x^3 + 6x^2 - 12x}{3x}$ =

$\frac{18a^2 + 12ab}{3a}$ =

$\frac{18x^2 + 8x + 6}{3}$ =

$\frac{20xy^5 - 15xy^2 + 30x^2y^4}{5xy}$ =

Practice:
Page 65 # (36-38 ace), 39

2.4 –E- Removing the common factor

A Factor is an integer that divides evenly into another number.

The factors of 6 are… _____

The factors of 24 are… _____

Why are there 60 seconds in a minute, why not 100?

Factors of 60: _____

Factors of 100: _____

60 has 12 factors.
100 has 9 factors.

The Babylonians realized 60 is more convenient for their number system! (More factors)

The Greatest Common Factor (GCF) of a polynomial: is the largest factor that divides evenly into each term.

- Factoring is the exact opposite of expanding.
- We expand a product and factor a sum.
- To factor by removing the Greatest Common Factor:

1. Find the GCF→ the gcf of the coefficients , and the gcf of the variables (for each variable it will be the one with the smallest exponent)
2. Find the second factor: divide each term in the polynomial by the GCF you found.
3. Always check by expanding.

Ex 1: Find the gcf

a) 8 , 16 , 40 _____

b) $6x^2$, $24x^3$, $12x^4$ _____

c) $28x^2y^2$, $14x^3y^2$, $21x^2y^3$ _____

d) $15a^6b^7$, $3a^3b^5$, $21a^6b^4$ _____

Ex 2: Factor by removing the gcf

a) $5x + 10y – 15$ **b) $12x^2 – 8x$**

Ex 3: Factor by pulling out the GCF

$4x+6=$ _____

$9x-15=$ _____

$6x^2+10x=$ _____

$49x^3y^2 - 21x^2y^2 + 14x^3y^3=$ _____

$2a^2b^2-6ab^3+4ab^2=$ _____

$x(x+3) + 2(x+3)=$ _____

Practice page 67 # 44 - 47

Practice:
P. 67 # 44 – 47